ECON5152 — Network Analysis

Week 7: Strategic and Economic Networks

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Reading for this week

- Necessary:
  - Jackson, *Social and Economic Networks*: Chapter 6
  - Newman, *Networks*: Chapter 13

Outline

1. Preferential attachment
2. Network formation
3. Linking utility and costs
4. Pairwise stability
5. Connections model
6. Applications
    - Social networks
    - Collaboration and trade networks

Preferential Attachment

- Before moving onto fully strategic networks, review one final important concept from random networks
- As mentioned previous, features of real-world networks include:
  - *Heterogeneous degrees*
  - High clustering
  - Communities
  - "Small world"
- Erdős–Rényi model predicts identically distributed degrees from a Binomial / Poisson distribution
- Configuration model fixes degree sequence beforehand
- Question: how to naturally obtain degrees that follow real-world behaviour?
- *Preferential attachment* models seek to address heterogeneous degrees

Preferential Attachment

- Key idea behind preferential attachment (and reason for the name): popularity
- Imagine a growing network where new nodes are added over time
- New nodes will want to connect to those who are already popular
- Idea developed as parallel to “rich get richer” in income data
- Observed in
  - Internet (WWW)
  - Social media
  - Brand reputation
  - Biological networks (e.g., DNA gene expression)

Preferential Attachment

- Probability any node $j \in \mathcal{N}$ links to node $i \in \mathcal{N}$ that has degree d_i:

$$\Pr(\text{Link } ij \text{ is formed}) = \Pr(A_{i,j} = 1) = \frac{d_i}{\sum_j d_j}$$

- Simple idea generates degree distributions following *power laws*

$$\Pr(d_i = k) = c \cdot k^{-\alpha}$$

- α is the power law coefficient governing how fast degrees grow
- c is just a constant depending on α

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- Simple idea generates degree distributions following *power laws*

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- α is the power law coefficient governing how fast degrees grow
- c is just a constant depending on α
- This simple idea reproduces:
  - Cumulative advantage
  - Visibility and reputation (brand) effects
  - Endogenous emergence of hubs

Power law degrees

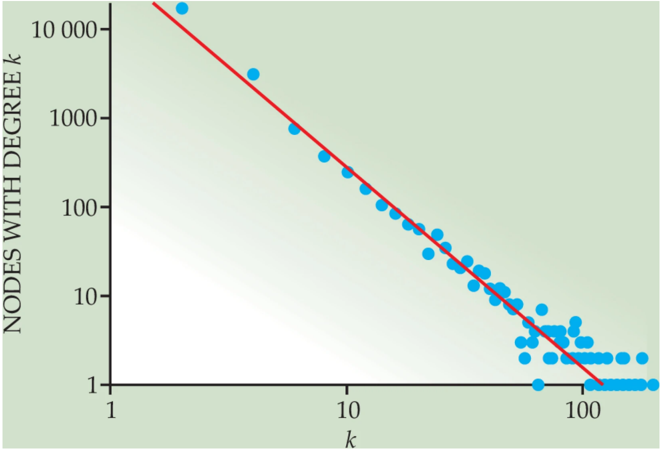

Figure: Degree distribution of the internet vs a power law with $\alpha \approx 2.1$ (Log scale)

Preferential Attachment Models

- *Price model* uses "rich get richer" idea to model degrees
- Grow a (directed) network, connecting new nodes based on existing degree
- Additional random steps to create connectedness ("small world")
- Citation network as main motivation
- Highly cited papers get cited more often by new papers
- Creates scientific reputation effects

Preferential Attachment Models

- *Price model* uses "rich get richer" idea to model degrees
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- Citation network as main motivation
- Highly cited papers get cited more often by new papers
- Creates scientific reputation effects
- *Barabási-Albert* is an undirected version
- Connect new nodes to exactly c others every step, based on degree
- See Newman, *Networks*, 2nd ed., Chapter 13 for more details and models

Network formation

Previous weeks:

- Networks as deterministic objects
- Networks as random realizations

This week:

- Networks as *choices*
- Links formed (or not) based on incentives
- Strategic decisions of nodes influence edges created

Key question:

Why does a particular network form?

Strategic Network Formation

- Nodes $\mathcal{N}$ represent agents
- Edges $\mathcal{E}$ represents a mutual relationship
  - Friendship
  - Purchase
  - Communication
- Agents evaluate costs and benefits of links
- Links can be costly to maintain
- Links of others are also beneficial to me indirectly
- Network structure is an *equilibrium outcome*
- How to define an equilibrium concept?

Costs and Benefits of Links

Each agent i chooses links to maximize:

$$u_i(\mathcal{G}) = \text{Benefits from network } \mathcal{G} \ - \ \text{Costs of links}$$

Key tradeoff:

- More links $\Rightarrow$ better access
- More links $\Rightarrow$ higher costs

Linking Utility

Connection benefits in the network can be decomposed into

- Direct benefits: neighbors
- Indirect benefits: neighbors of neighbors (of neighbours...)

Distance in the network matters:

- Information decays with distance
- Longer paths provide weaker benefits

Simple Model of Network Benefits

Let:

- $d_{ij}(\mathcal{G})$ = network distance between i and j
- $\delta \in (0, 1)$ = decay parameter

Benefit from j to i:

$$u_i(\mathcal{G}) = \delta^{d_{ij}(\mathcal{G})}$$

Intuitively: closer agents are weighed more strongly

Assume:

- Each direct link costs $c > 0$
- Cost is paid by both endpoints

Agents balance:

Indirect reach vs. direct cost

Networks as Strategic Objects

Important distinction:

- Random graphs: probability determines links
- Strategic models: incentives determine links

Outcomes depend on:

- Utility of linking
- Cost level
- Decay of benefits
- Links of others (externalities)

Pairwise Stability — Motivation

A network is stable if:

- No agent wants to sever an existing link
- No pair of agents wants to add a missing link

Stability is local and incentive-based.

Pairwise Stability — Definition

A network $\mathcal{G}$ is *pairwise stable* if:

- For any existing link $(i, j) \in \mathcal{G}$:

$$u_i(\mathcal{G}) \geq u_i(\mathcal{G} - ij) \quad \text{and} \quad u_j(\mathcal{G}) \geq u_j(\mathcal{G} - ij)$$

- For any absent link $(i, j) \notin \mathcal{G}$:

$$\text{if } u_i(\mathcal{G} + ij) > u_i(\mathcal{G}) \Rightarrow u_j(\mathcal{G} + ij) < u_j(\mathcal{G})$$

- $\mathcal{G} - ij$ means removing link (i, j) and $\mathcal{G} + ij$ means adding it
- Existing links persist only if both benefit
- New links form only with mutual consent
- Key intuition: No unilateral profitable deviations

Pairwise Stability

- Simple and tractable
- Appropriate when links require consent
- Captures decentralized formation

Pairwise Stability

- Simple and tractable
- Appropriate when links require consent
- Captures decentralized formation
- Not necessarily efficient
- Other equilibrium concepts exist
- More specialized in the literature

Connections Model

Introduced by Jackson & Wolinsky, 1996:

- Agents benefit from being connected
- Benefits decay with distance
- Direct links are costly
- A canonical model of strategic network formation

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Introduced by Jackson & Wolinsky, 1996:

- Agents benefit from being connected
- Benefits decay with distance
- Direct links are costly
- A canonical model of strategic network formation
- Agent i's utility:

$$u_i(\mathcal{G}) = \sum_{j \neq i} \delta^{d_{ij}(\mathcal{G})} - c \cdot d_i(\mathcal{G})$$

  - δ: decay factor
  - c: cost per link

Extreme Networks

Benchmark structures:

- Empty network
- Complete network
- Star network (additional incentives)
  - One central node
  - All others connect only to the center
  - Central agent bears many costs
  - Peripheral agents free-ride
- Their stability depends on model parameters
- c and δ govern behaviour and equilibrium outcomes

Inefficiency vs Stability of Network Formation

- Efficient network maximizes total surplus
- Stable network maximizes individual incentives
- These objectives need not coincide
- Possible outcomes:
  - Too few links
  - Too many links
  - Wrong structure
- Externalities matter! (Links generated by others)
- Think of following thought experiment:
  - Start from a complete and trim it until stable
  - Start from empty and connect it until stable
  - Both yield alternative equilibria and are used to understand properties

Applications

Social Networks

- Friendship formation
- Collaboration networks
- Information sharing
- $\implies$ Links reflect strategic tradeoffs

Trade and Production Networks

- Firms choose suppliers
- Redundancy vs cost minimization
- Indirect access matters
- $\implies$ Strategic formation complements IO analysis

Key Takeaways

- Week 4: propagation given a network
- Week 5–6: random network structure
- Week 7: incentives generate structure
- Structure, randomness, and incentives interact in the real world

Key Takeaways

- Week 4: propagation given a network
- Week 5–6: random network structure
- Week 7: incentives generate structure
- Structure, randomness, and incentives interact in the real world
- Networks can be equilibrium outcomes
- Costs and indirect benefits shape structure
- Pairwise stability is a basic equilibrium notion
- Efficient and stable networks may differ