ECON5152 — Network Analysis

Week 5: Random Graphs

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Reading for this week

- Necessary:
  - Newman, *Networks*: Chapter 11
  - Jackson, *Social and Economic Networks*: Sections 4.1.1, 4.2.1–4.2.5
- Optional:
  - Jackson, *Social and Economic Networks*: Remaining sections of Chapter 4 and Chapter 5

Outline

1. Clustering
2. Motivation
3. Simplest random graph (Erdős-Rényi model)
4. Average properties
5. Large random graphs (Poisson model)
6. Phase transitions
7. Limitations of random graphs

Clustering

- Before moving onto random networks, we define an important property
- *Clustering* refers to the amount of local groups in a network
- We know that real-world networks are characterized by:
  - Local clusters
  - Large group structure
  - Sparsity: density is usually small
  - Heterogeneous degrees: many nodes with small degrees and few with very large degrees (also called "skewed degree distribution")
- Examples:
  - Social networks
  - Firm locations
  - Internet

Clustering Example

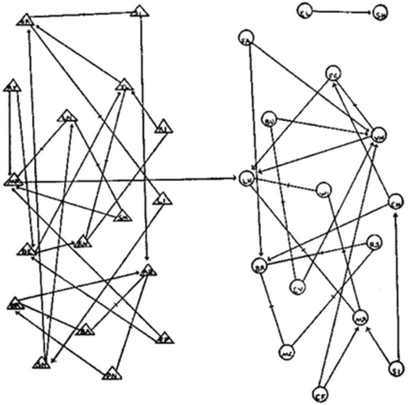

Figure: Friendship between boys (triangles) and girls (circles) in a 1930's schoolchildren class

Local Clustering

- How to measure the level of clustering around a single node?
- Simple measure: if a node has two links, how likely are they to be linked to each other?

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Local clustering coefficient

The *local clustering coefficient* $C_i(\mathcal{G})$ of node $i \in \mathcal{N}$ in network $\mathcal{G}$ is the ratio of the number of pairs of i's links that are themselves connected to the total number of pairs of i's links.

- Look at any pair of links for i, say j and k: for these, $A_{i,j} = A_{i,k} = 1$
- Are j and k connected? In other words, is $A_{j,k} = 1$?
- Count the number of pairs for which this is true
- Divide the count by the total number of pairs of i's links

Local Clustering

- Let $d_i = \sum_{j=1}^{n} A_{i,j}$ be the degree (total links of i)
- Let $n_i(\mathcal{G})$ be the total pairs of i's links (depends on network structure)
  - Directed & self-loops $\rightarrow n_i(\mathcal{G}) = d_i^2$
  - Directed & no self-loops $\rightarrow n_i(\mathcal{G}) = d_i(d_i - 1)$
  - Undirected & self-loops $\rightarrow n_i(\mathcal{G}) = d_i(d_i + 1)/2$
  - Undirected & no self-loops $\rightarrow n_i(\mathcal{G}) = d_i(d_i - 1)/2$
- Number of pairs of i's links that are also connected in $\mathcal{G}$:

$$t_i(\mathcal{G}) := f(\mathcal{G}) \cdot \sum_{j \neq i}^{n} \sum_{k \neq i, k \neq j}^{n} A_{i,j} A_{i,k} A_{j,k} \text{ where } f(\mathcal{G}) = \begin{cases} 1/2 & \text{if } \mathcal{G} \text{ undirected} \\ 1 & \text{if } \mathcal{G} \text{ directed} \end{cases}$$

- Local clustering coefficient: $C_i(\mathcal{G}) = t_i(\mathcal{G})/n_i(\mathcal{G})$ $\left[C_i(\mathcal{G}) = 0 \text{ if } n_i(\mathcal{G}) = 0\right]$
- Ratio of number of triangles around i to number of triples with i or $n_i(\mathcal{G})$

Local Clustering Example

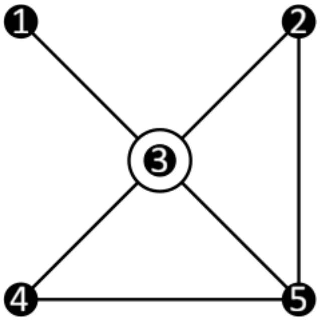

Figure: Network around central node $i = 3$

- Pairs around 3: $\{1,2\}$, $\{1,4\}$, $\{1,5\}$, $\{2,4\}$, $\{2,5\}$, $\{4,5\}$
- Pairs around 3 who are also connected: $\{2,5\}$, $\{4,5\}$
- In our notation: $t_3(\mathcal{G}) = 2$ and $n_i = 4(3)/2 = 6$
- Local clustering coefficient: $C_3(\mathcal{G}) = 2/6 = 1/3$

Local Clustering Example

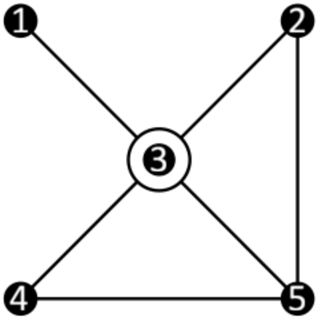

Figure: Network around central node $i = 3$

Equivalence # 1: number of triangles centred at i divided by $n_i(\mathcal{G})$:

$$C_3(\mathcal{G}) = \frac{2}{6} = \frac{1}{3}$$

Equivalence # 2: for each neighbour of $i = 3$, count the number of links to other neighbours and divide by twice the total

$$C_3(\mathcal{G}) = \frac{\overbrace{0}^{\text{From 1}} + \overbrace{1}^{\text{From 2}} + \overbrace{1}^{\text{From 4}} + \overbrace{2}^{\text{From 5}}}{2 \times 6}$$

$$= \frac{4}{12} = \frac{1}{3}$$

Global Clustering

Clustering coefficient

The *clustering coefficient* $C(\mathcal{G})$ of network $\mathcal{G}$ is the ratio of the total number of triangles over the total number of triples around each node:

$$C(\mathcal{G}) := \frac{\sum_{i=1}^{n} t_i(\mathcal{G})}{\sum_{i=1}^{n} n_i(\mathcal{G})} = \frac{\sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{k=1}^{n} A_{i,j} A_{i,k} A_{j,k}}{\sum_{i=1}^{n} n_i(\mathcal{G})}$$

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Average clustering coefficient

The *average clustering coefficient* $\bar{C}(\mathcal{G})$ of network $\mathcal{G}$ is the average of the local clustering coefficients:

$$\bar{C}(\mathcal{G}) := \frac{1}{n}\sum_{i=1}^{n} C_i(\mathcal{G}) = \frac{1}{n}\sum_{i=1}^{n} \frac{t_i(\mathcal{G})}{n_i(\mathcal{G})}$$

Global Clustering Example

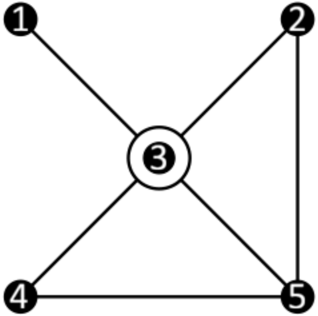

Figure: Consider all nodes $\mathcal{N} = \{1, 2, 3, 4, 5\}$

Each local clustering coefficient:

$$t_1(\mathcal{G}) = 0, n_1(\mathcal{G}) = 0 \implies C_1(\mathcal{G}) = 0$$
$$t_2(\mathcal{G}) = 1, n_2(\mathcal{G}) = 1 \implies C_2(\mathcal{G}) = 1$$
$$t_3(\mathcal{G}) = 2, n_1(\mathcal{G}) = 6 \implies C_3(\mathcal{G}) = 1/3$$
$$t_4(\mathcal{G}) = 1, n_4(\mathcal{G}) = 1 \implies C_4(\mathcal{G}) = 1$$
$$t_5(\mathcal{G}) = 2, n_5(\mathcal{G}) = 3 \implies C_5(\mathcal{G}) = 2/3$$

Global and average clustering coefficients:

$$C(\mathcal{G}) = \frac{0+1+2+1+2}{0+1+6+1+3} = \frac{6}{11}$$
$$\bar{C}(\mathcal{G}) = \frac{1}{5}\left(0 + 1 + \frac{1}{3} + 1 + \frac{2}{3}\right) = \frac{3}{5}$$

Motivation

- So far, networks are *given objects*
- Fix n nodes $\mathcal{N}$ and edges $\mathcal{E}$ to obtain graph $\mathcal{G}$
- Adjacency matrix $\mathbf{A}$ of size $n \times n$ is fixed given graph $\mathcal{G}$
- Study properties of fixed $\mathcal{G}$ and $\mathbf{A}$

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- Study properties of fixed $\mathcal{G}$ and $\mathbf{A}$

Question for you

Where does the graph $\mathcal{G}$ come from?

- How is $\mathcal{G}$ formed in the real world?
- What are the determinants behinds links in $\mathcal{G}$?
- Why are some links formed and not others?

Random Networks

- For now, abstract away from all *strategic* choices of $\mathcal{G}$
  - Preferences
  - Popularity
  - Sociability
- Consider what happens if links are formed entirely at random
- *Random graphs* are simple benchmarks for *network formation*
  - Separate structural patterns from noise
  - Study typical vs exceptional networks
  - Make probabilistic statements about networks
  - Study properties of large networks

Random Networks

- Let $\mathfrak{G}$ represent the collection of *all* possible networks

Random graph model

A *random graph model* is a probability distribution P over $\mathfrak{G}$.

- A random graph $\mathcal{G}$ is a realization of P from all possible $\mathfrak{G}$
- Denoted as $\mathcal{G} \sim P$
- Properties of a random $\mathcal{G}$ are themselves random and depend on model P
- E.g., number of links, density, path lengths, centrality, etc.

Simplest random graphs

Fix a set of n nodes $\mathcal{N} := \{1, \ldots, n\}$

Question for you

- How to choose a network $\mathcal{G}$ at random?
- Equivalently, how to choose $\mathcal{E}$ at random?

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1. Method 1 (P_1): pick a given number of edges m to form at random

Simplest random graphs

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Question for you

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- Equivalently, how to choose $\mathcal{E}$ at random?

1. Method 1 (P_1): pick a given number of edges m to form at random
2. Method 2 (P_2): form any link independently with probability p

Method 1: Fixed number of edges

- Let m be the fixed number of edges
- Let N be the number of potential links, e.g., $n(n-1)/2$ if undirected $\mathcal{G}$
- N depends on network structure (see definition of n_i and Week 2 slides)
- Extreme cases:
  1. For $m = 0$, network is empty
  2. For $m = n$, recover complete network
- All values in-between provide fully random graphs
- For fixed n, as m increases, graph becomes denser

Method 1: Example

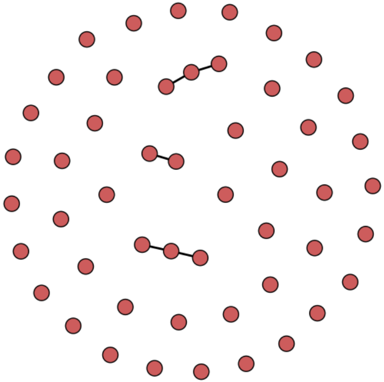


Figure: Random graph with $n = 50$ nodes and $m = 5$ edges

Method 1: Example

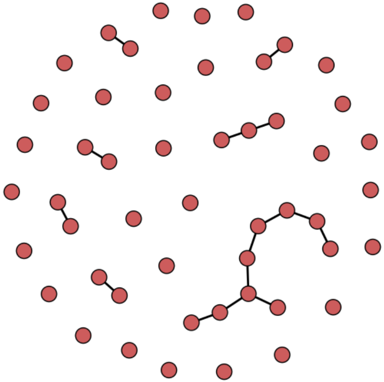


Figure: Random graph with $n = 50$ nodes and $m = 15$ edges

Method 1: Example

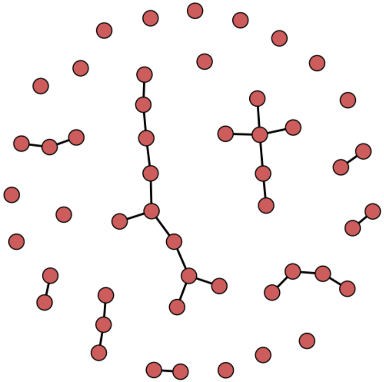


Figure: Random graph with $n = 50$ nodes and $m = 25$ edges

Method 1: Example

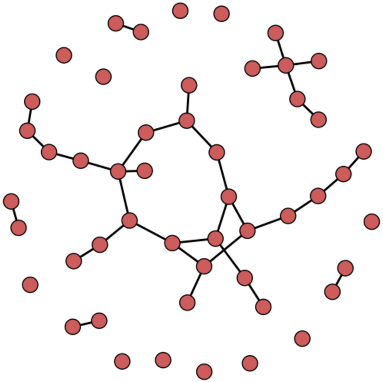


Figure: Random graph with $n = 50$ nodes and $m = 35$ edges

Method 1: Example

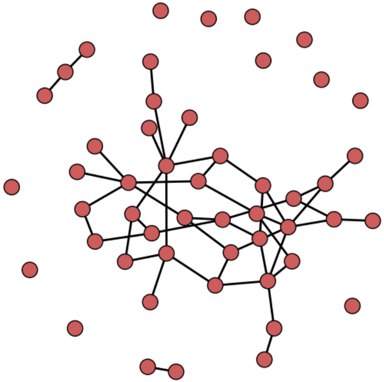


Figure: Random graph with $n = 50$ nodes and $m = 48$ edges

Method 2: Fixed probability of linking

- Let p be the fixed probability of creating an edge
- Extreme cases:
  1. For $p = 0$, network is empty
  2. For $p = 1$, recover complete network
  3. For $p = 1/2$, *uniform probability over networks*
- All values in-between provide fully random graphs
- For fixed n, as p increases, graph becomes denser
- Clear parallel of p (linking probability) and m (number of edges)

Method 2: Example

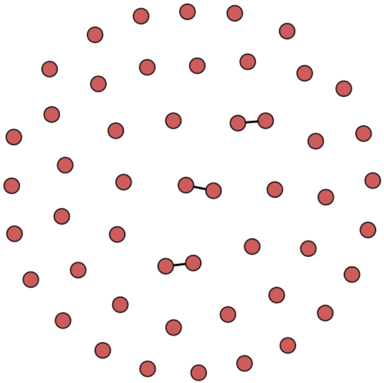


Figure: Random graph with $n = 50$ nodes and $p = 5/50$ linking probability

Method 2: Example

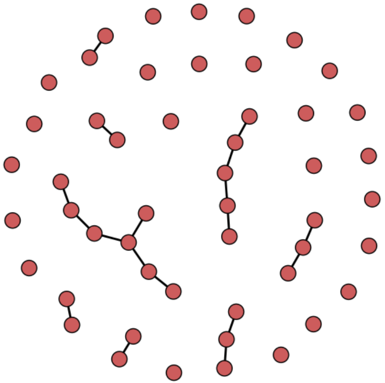


Figure: Random graph with $n = 50$ nodes and $p = 15/50$ linking probability

Method 2: Example

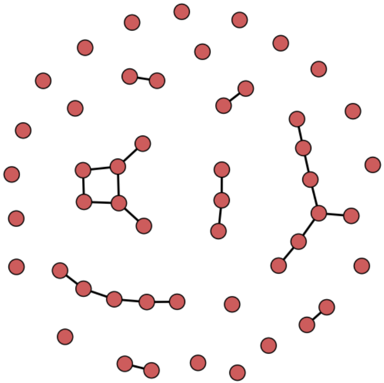


Figure: Random graph with $n = 50$ nodes and $p = 25/50$ linking probability

Method 2: Example

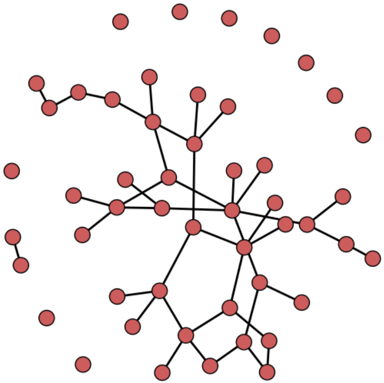


Figure: Random graph with $n = 50$ nodes and $p = 35/50$ linking probability

Method 2: Example

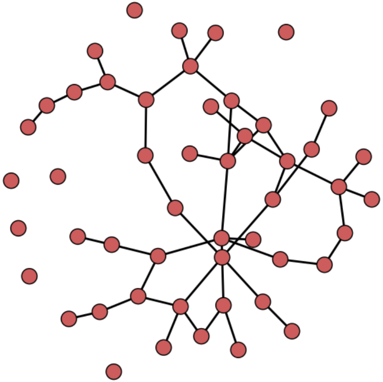


Figure: Random graph with $n = 50$ nodes and $p = 48/50$ linking probability

The Erdős–Rényi Model

- Two equivalent formulations of the *Erdős–Rényi model*

1. $\mathcal{G}(n, m)$: graph chosen uniformly with exactly m edges
2. $\mathcal{G}(n, p)$: each edge included independently with probability p

- Correspond to random graph models P_1 and P_2, respectively
- Generally focus on $\mathcal{G}(n, p)$ as it is more tractable
- Models $\mathcal{G}(n, m)$ and $\mathcal{G}(n, p = m/N)$ behave similarly (see previous graphs)
- A *realization* of these models is a single network out of the 2^N possibilities

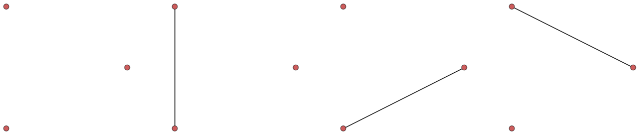
The Erdős–Rényi Model

- Let $\mathfrak{G}_m$ be all the networks that have m edges
- If $\mathcal{G} \in \mathfrak{G}_m$ (i.e., $\mathcal{G}$ has exactly m edges) then

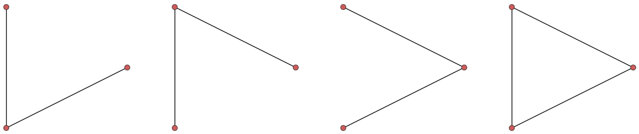
$$P_1(\mathcal{G}) = \frac{1}{\binom{N}{m}} = \frac{m!(N-m)!}{N!}$$
$$P_2(\mathcal{G}) = p^m \cdot (1-p)^{N-m}$$

- $\binom{N}{m}$ counts the number of ways to choose m edges out of the total N
- Both processes assign different probabilities to the same graph
- However, both are similar when n is large and $p \approx m/N$
- It can be shown that any property satisfied by P_1 is also satisfied by P_2
- Note $p = 1/2 \implies P_2(\mathcal{G}) = 1/2^N$ (uniform over all 2^N possibilities!)

Erdős–Rényi Model Example



(a) $\mathcal{G}_0$ (b) $\mathcal{G}_{1,1}$ (c) $\mathcal{G}_{1,2}$ (d) $\mathcal{G}_{1,3}$



(e) $\mathcal{G}_{2,1}$ (f) $\mathcal{G}_{2,2}$ (g) $\mathcal{G}_{2,3}$ (h) $\mathcal{G}_3$

Graph	P_1	P_2
$\mathcal{G}_0$	1	$(1-p)^3$
$\mathcal{G}_{1,1}$	$1/3$	$p(1-p)^2$
$\mathcal{G}_{1,2}$	$1/3$	$p(1-p)^2$
$\mathcal{G}_{1,3}$	$1/3$	$p(1-p)^2$
$\mathcal{G}_{2,1}$	$1/3$	$p^2(1-p)$
$\mathcal{G}_{2,2}$	$1/3$	$p^2(1-p)$
$\mathcal{G}_{2,2}$	$1/3$	$p^2(1-p)$
$\mathcal{G}_3$	1	p^3

Table: Probabilities assigned to undirected graphs with $n = 3$ and $N = 3$

Erdős–Rényi Model Example

Graph	$p = 0$	$p = 1/3$	$p = 1/2$	$p = 2/3$	$p = 1$
$\mathcal{G}_0$	1	8/27	1/8	1/27	0
$\mathcal{G}_{1,1}$	0	4/27	1/8	2/27	0
$\mathcal{G}_{1,2}$	0	4/27	1/8	2/27	0
$\mathcal{G}_{1,3}$	0	4/27	1/8	2/27	0
$\mathcal{G}_{2,1}$	0	2/27	1/8	4/27	0
$\mathcal{G}_{2,2}$	0	2/27	1/8	4/27	0
$\mathcal{G}_{2,2}$	0	2/27	1/8	4/27	0
$\mathcal{G}_3$	0	1/27	1/8	8/27	1
Total	1	1	1	1	1

Table: Probabilities assigned by $\mathcal{G}(n, p)$ (P_2) with different p to undirected graphs where $n = 3$ and $N = 3$

Average or Expected Properties

- Individual realizations of random graphs are difficult to analyse
- A collection of realizations is easier to analyse
- In particular, consider *averages* or *expectations* over process P
- For any network statistic $S(\mathcal{G})$, the expected value over P is

$$\mathbb{E}_P[S] = \sum_{\mathcal{G} \in \mathfrak{G}} S(\mathcal{G}) \cdot P(\mathcal{G})$$

- Calculate statistic for each network and take weighted average (weights is probability attached to each network)

Average or Expected Properties

- Focus on results for $\mathcal{G}(n, p)$ (Erdős–Rényi with probability p)
- Denote the total number of edges for network $\mathcal{G}$ as $E(\mathcal{G})$
- There are $\binom{N}{m}$ networks of size m: ways to choose m edges out of total N
- For all $\mathcal{G}$ with m edges ($\mathcal{G} \in \mathfrak{G}_m$), $P(\mathcal{G}) = p^m(1-p)^{N-m}$ is identical
- Probability of sampling a network $\mathcal{G}$ with m edges:

$$P(\mathcal{G} \in \mathfrak{G}_m) = \binom{N}{m} p^m (1-p)^{N-m}$$

- Binomial distribution with m successes, N trials and probability p!

Expected Number of Edges

- Binomial distribution of m edges out of N possible with linking probability p
- Let $E(\mathcal{G})$ denote the total number of edges
- Expected total number of edges:

$$\mathbb{E}[E] = \sum_{\mathcal{G} \in \mathfrak{G}} S(\mathcal{G}) \cdot P(G) = \sum_{m=0}^{N} \sum_{\mathcal{G} \in \mathfrak{G}_m} E(\mathcal{G}) \cdot p^m (1-p)^{N-m}$$
$$= \sum_{m=0}^{N} m \cdot \binom{N}{m} \cdot p^m (1-p)^{N-m}$$
$$= N \cdot p$$

- Last line follows from Binomial distribution
- Intuitively: each link formed with probability p from all possible N links

Average Degree and Clustering

- Recall d_i is the degree of any node $i \in \mathcal{N}$
- Using similar intuition: what is $\mathbb{E}[d_i]$?

Average Degree and Clustering

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- Any link formed with probability p
- Any node can form at most $n-1$ links to others
- Consider possible values for degrees $k \in \{0, \ldots, n-1\}$

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- Any node can form at most $n-1$ links to others
- Consider possible values for degrees $k \in \{0, \ldots, n-1\}$
- $P(d_i = k) = \binom{n-1}{k} p^k (1-p)^{n-1-k}$, another Binomial!
- Expected degree: $\mathbb{E}[d_i] = (n-1) \cdot p$
- Expectation from a Binomial with $n-1$ trials and probability p
- Same for all nodes $i \in \mathcal{N}$

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- Same for all nodes $i \in \mathcal{N}$
- **Exercise**: with similar intuition, argue $\mathbb{E}[C] = p$ (global clustering coefficient is also p)

Degree Distribution

- We have seen that degrees $d_1, \ldots, d_n$ are an important statistic
- Collection of degrees is called *degree distribution*
- Degree distribution of $\mathcal{G}(n, p)$ model with probability p fully characterized
- Degree distribution identical for all nodes
- In Erdős–Rényi random graphs, no node is special

Large Graphs

- It is difficult to study more properties of (Erdős–Rényi or ER) random graphs
- Much simpler to study what happens as n grows
- With $n \to \infty$, we study properties of *large graphs*
- As n is the number of nodes, more nodes means more information
- *Law of large numbers* guarantees typical large network looks like the average
- Still, no specific realization is exactly average!

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- As n is the number of nodes, more nodes means more information
- *Law of large numbers* guarantees typical large network looks like the average
- Still, no specific realization is exactly average!
- Important distinction: how does p behave as $n \to \infty$?
  - *Sparse* networks: $p \to 0$ (less connections form)
  - *Dense* networks: $p \to 1$ (most connections form)
  - If p remains constant: random structure

Degree Distribution

- Let us revisit the degree d_i for any node $i \in \mathcal{N}$
- We now let $n \to \infty$ to reflect large networks
- Under the Erdős–Rényi model with probability p we had

$$P(d_i = k) = \binom{n-1}{k} p^k (1-p)^{n-1-k}$$

- Call the average
- We can show (see 11.7 to 11.10 in Newman) that

$$P(d_i = k) \underset{n \to \infty}{\to} e^{-(n-1)\cdot p} \frac{[(n-1)\cdot p]^k}{k!}$$

- This is a Poisson distribution over the degree for large n

Poisson random graphs

Poisson random graphs

Poisson random graphs have degree d_i for node $i \in \mathcal{G}$ behaving as

$$\text{Poisson}(d_i = k) = e^{-(n-1)\cdot p} \frac{[(n-1)\cdot p]^k}{k!}$$

- Poisson random graphs are those with degree distributions distributed according to the Poisson distribution
- Important fact: $\mathbb{E}[d_i] = (n-1) \cdot p = \text{Var}(d_i)$
- *Poisson random graphs are the limits of Erdős–Rényi graphs*

Paths and Connectivity

With large networks, we can now ask probabilistically:

- When can we expect links to form?
- When do groups start forming?
- When does a "giant component" (formed by many nodes) start forming?
- Is the graph connected?
- What is the average length of shortest paths?

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All the answers depend on the behaviour of p as n grows

Paths and Connectivity

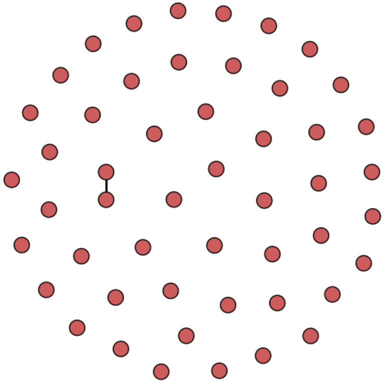

Figure: Random graph with $n = 50$ nodes and $p = 0.0004$ linking probability

Paths and Connectivity

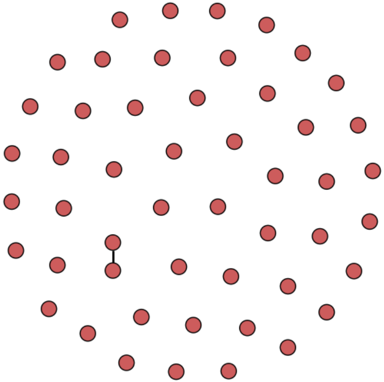

Figure: Random graph with $n = 50$ nodes and $p = 0.003$ linking probability

Paths and Connectivity

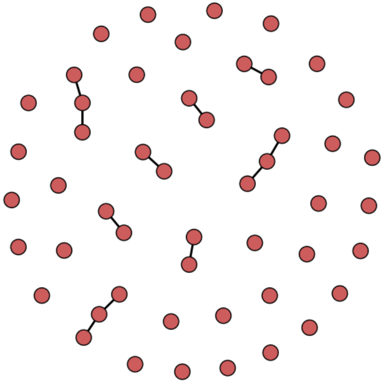

Figure: Random graph with $n = 50$ nodes and $p = 0.01$ linking probability

Paths and Connectivity

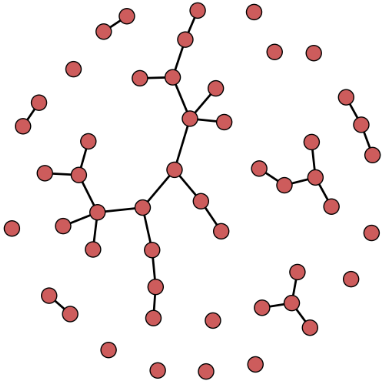

Figure: Random graph with $n = 50$ nodes and $p = 0.02$ linking probability

Paths and Connectivity

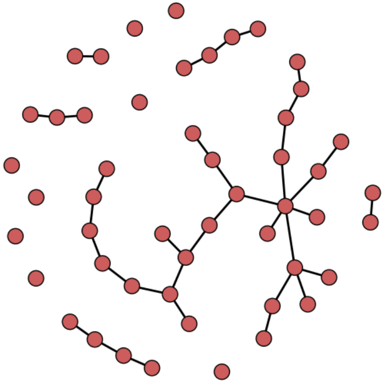

Figure: Random graph with $n = 50$ nodes and $p = 0.03$ linking probability

Paths and Connectivity

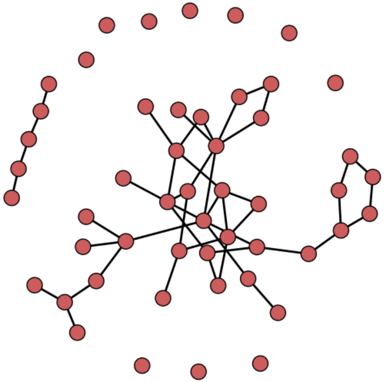

Figure: Random graph with $n = 50$ nodes and $p = 0.05$ linking probability

Paths and Connectivity

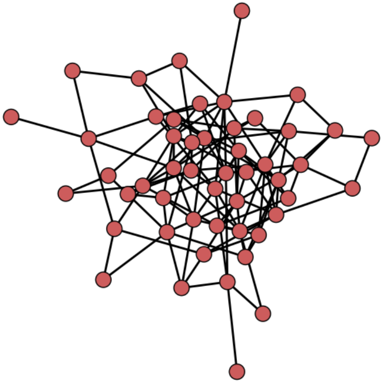

Figure: Random graph with $n = 50$ nodes and $p = 0.08$ linking probability

Paths and Connectivity

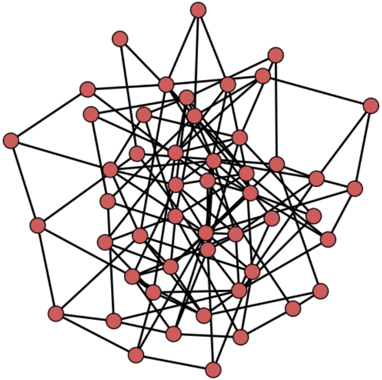

Figure: Random graph with $n = 50$ nodes and $p = 0.1$ linking probability

Phase Transitions

- There are thresholds $\tau(n)$ for p, such that:
  - A property forms with probability 0 below the threshold or $p < \tau(n)$
  - Property forms with probability 1 above the threshold or $p > \tau(n)$
- As p grows for a fixed n, *phase transition* from one state to another
- In one state, something guaranteed not to happen
- In other state, it is guaranteed to happen

Phase Transitions

A list of phase transitions for large graphs (verify with previous figures):

1. At $p = \tau(n) = 1/n^2$, the first link starts to form
2. At $p = \tau(n) = 1/n^{3/2}$, the first component with more than two nodes
3. At $p = \tau(n) = 1/n$ cycles and a "giant component" emerge
4. At $p = \tau(n) = \log(n)/n$, the network becomes connected

Small changes to p can cause large structural shifts to the resulting networks.

Random Graphs as Benchmarks

- If we understand random graphs, we know what networks and their properties look like on average
- Can help us understand benchmark scenarios
- Can then also compare real-world networks to benchmarks
  - Excess clustering
  - Heavy-tailed degree distributions
  - Community structure
- If ER cannot explain it, additional structure is present

Limitations of ER Graphs

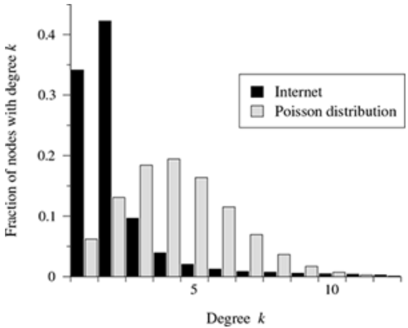

Figure: Degrees of internet connection between "domains" and the random graph prediction (Figure 11.8 in Newman)

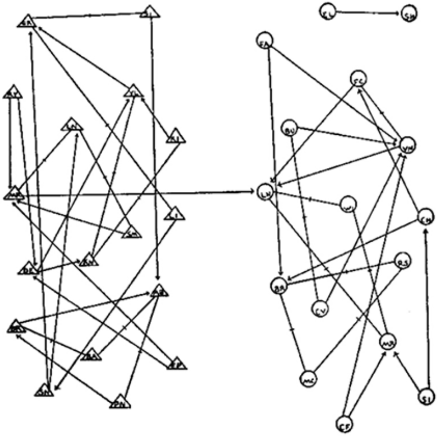

Figure: Clustering in social networks

Limitations of ER Graphs

ER graphs systematically lack fundamental structure

- No degree heterogeneity
- No communities
- No core–periphery
- No closure of triangles beyond chance

ER graphs cannot capture features of real-world networks

- Heavy-tailed degrees
- High clustering
- Modularity

Key Takeaways

- Random graphs model networks probabilistically
- Erdős–Rényi graphs provide a useful benchmark
- Expectations (typical or average behaviour) and realizations differ
- Characterize expectations and large graphs
- Limitations motivate richer models!