ECON5152 — Network Analysis

Week 10: Advanced Topics and Complex Networks

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Outline

Motivation

- Introduce a wide range of *frontier network tools*
- Material in course up to now focuses on:
  - Dyadic tools
  - Single edges
  - Single networks
  - Static environments
- And all the learning that comes with them
- Goal: *Extend beyond these limitations*

Dynamic and Temporal Networks

- Begin with simple extension: *dynamics* or *time*
- Time has an order: $t = 1, 2, 3, \ldots, T-2, T-1, T$
- Nodes represent agents present at time t: $\mathcal{N}_t$
- Edges can occur between nodes at time t: $\mathcal{E}_t$
- Links may or may not be present *across time*
  1. Dynamic networks (same network across time periods)
  2. Time networks (flow of links follows a direction)
  3. Temporal networks and timing causality (links within and across time)

Simple Example: Two Time Periods

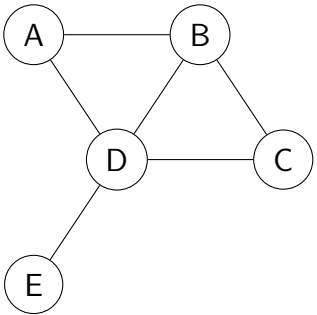

Figure: Network at time $t = 1$

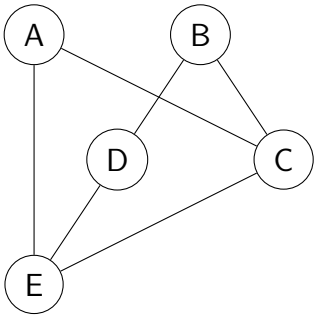

Figure: Network at time $t = 2$

Example: Trade Networks

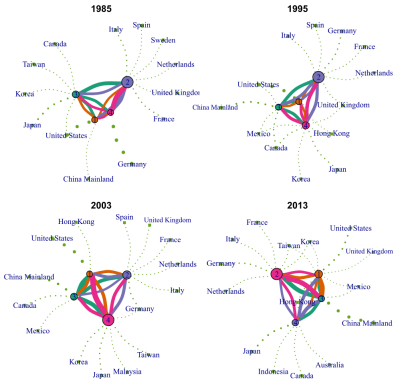

Figure: Latent trade hubs across time (Chen and Chen, 2023, doi.org/10.6339/22-JDS1065)

Multiplex Networks

- Start to allow *multiple types of connections*
- *Multiplex* networks are special cases of *multilayers*
- *Same* nodes with different type of connections
- No connections across nodes
- Dynamic networks are multiplex networks!

Example

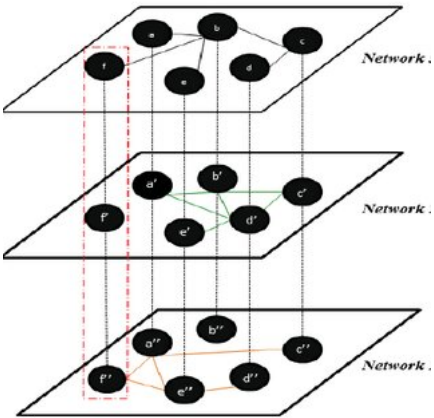

Figure: Multiplex connections

Multilayer Networks

- Allow *any type* of multiple connections
- *Multilayer* networks link within and across layers
- Potentially *different* nodes with different type of connections
- Temporal and causal networks are multilayer networks!

Example

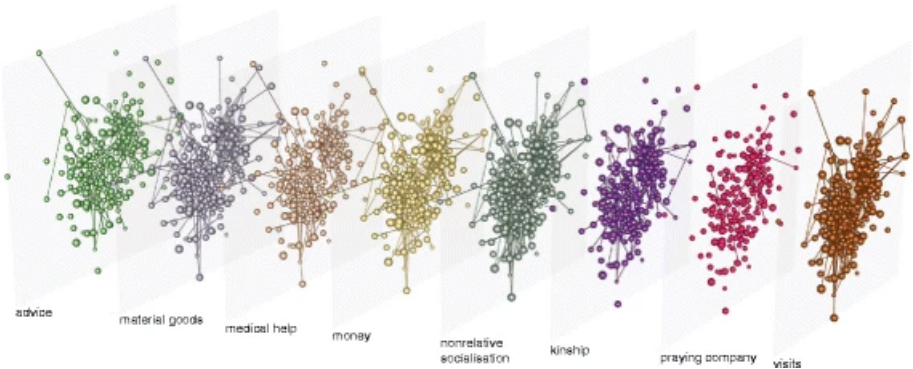

Figure: Multilayer social network (Omodei and Arenas, 2016, doi.org/10.1007/s41109-016-0016-x)

Hypergraphs

- Allow for more than *dyadic* (pair) structure
- *Hypergraphs* allow for edges *between more than two nodes*
- Simple example: families and teams
- Alternative representations of hypergraphs as standard graphs exist

Example

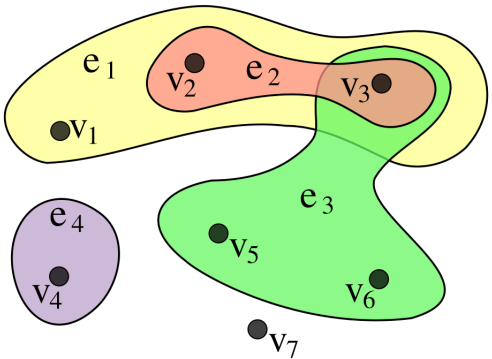

Figure: Hypergraph with 7 vertices (nodes) and 4 multi-edges

Real Examples

- Board of directors
- Members of congress / parliament
- Joint ventures:
  - Actors
  - Bands
  - Co-authors

Neural Networks

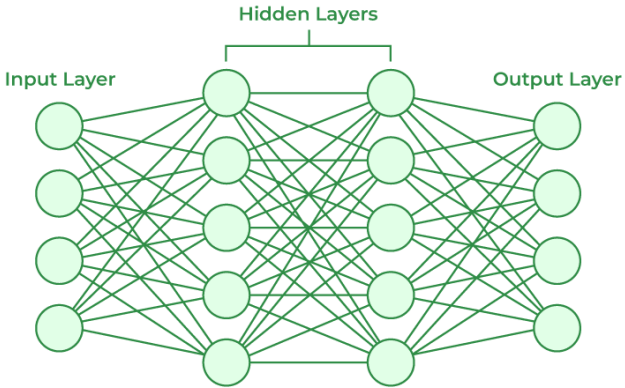

Figure: Neural network architecture

Neural Network Complexity

- n nodes per layer and $L+1$ layers
- Layer 0 is input, layer $L+1$ is output
- Parameters per layer:

$$n_p = (n+1)n$$

- Total parameters:

$$(n+1)n\frac{L(L+1)}{2}$$

- Growth:
  - Quadratic in nodes (n)
  - Quadratic in layers (L)

Scaling and LLMs

- Increase dimensionality of inputs (tokens)
- Increase depth (layers)
- Deep learning $\implies$ stacking transformations
- See GPT-3 example next

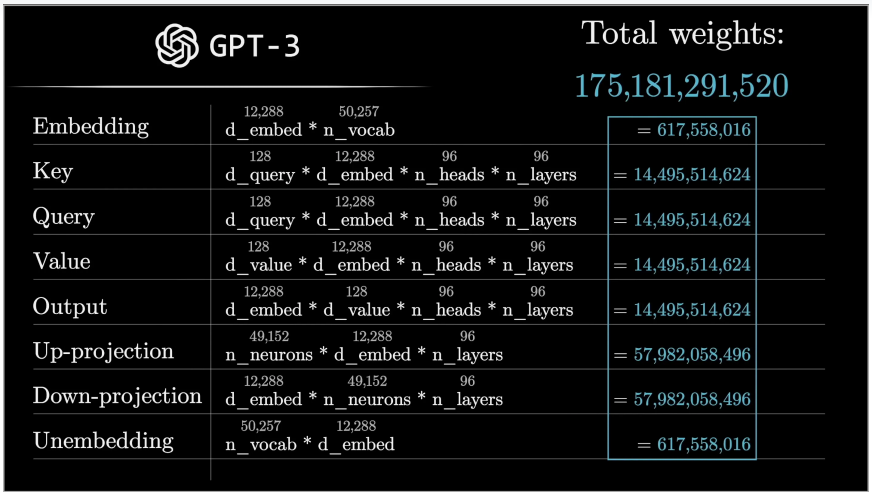

Figure: Number of parameters in Chat-GPT 3.0

Causality Networks

- Encode knowledge relationships
- Nodes are *variables* or *events*
- Edges represent *causal links* between nodes
- Structure depends on problem
- Ranges from simple graphs to multi-layers!

Simple Example: Directed Acyclical Graph (DAG)

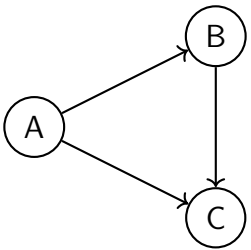

Figure: DAG structure: A = Ability, B = Test scores, C = Wages

Complex Causality Network

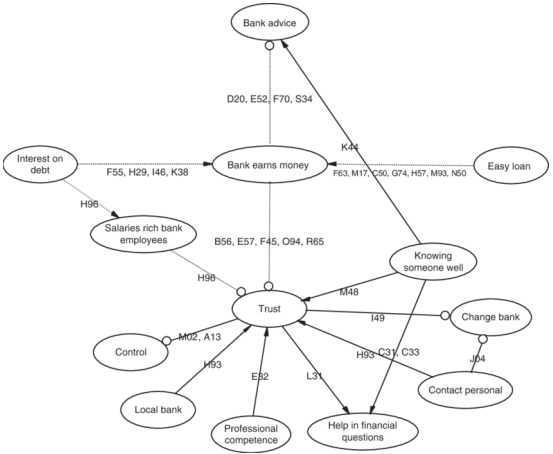

Figure: Causal decisions in managerial decisions